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OPTIMIZATION OF TECHNICAL SYSTEM RELIABILITY TAKING INTO ACCOUNT RESOURCE MINIMIZATION

This article explores optimizing the reliability of redundant technical systems through theoretical modeling, numerical methods, and economic optimization. It highlights balancing reliability and costs, critical thresholds, and strategies for resource allocation. Numerical methods identify critical zones in structures, supporting targeted reinforcement for enhanced performance and sustainability under varying operational conditions.

Keywords: *structural system reliability, resource efficiency, redundancy.*

Introduction

To ensure the required level of probability of failure-free operation of a redundant technical system (TS), for example, a ring water supply system, it is important not only to determine, but also to maintain the required reliability indicators for each element and subsystem that are part of it. This approach allows not only to set specific target values for individual components, but also to guarantee the coordinated failure-free operation of the entire system as a whole.

Particular attention should be paid to the optimal distribution of reliability among individual elements and subsystems. This task requires a systemic approach that takes into account the complexity of the TS structure and ensures that the required level of reliability is achieved with minimal costs. In practical terms, this means that increasing the reliability of the system's constituent elements should occur in such a way as not only to meet the specified parameters, but also to optimize resource and financial costs. Thus, the path to achieving high reliability of TS with a complex structure consists in a phased analysis of each element, assessing its impact on the overall network failure-free operation, and implementing measures that ensure the effective distribution of resources to achieve the target level of reliability.

For the required level of reliability of the TS while minimizing the costs associated with ensuring this reliability, it is important to find the optimal balance between the reliability indicators of the entire system and the costs of each of its elements or subsystems. This involves careful analysis and design to ensure a high level of network resilience to failures, while avoiding excessive costs that may be associated with the introduction of additional components or backup subsystems.

The concept of "failure" [1, 2, 4] is used to assess the reliability of the TS. It defines a situation when a consumer does not receive a target product, for example, water transported by the network. For the consumer, this is perceived as a system failure. However, the supplier may interpret this situation differently. If the target product is available to other consumers, such a situation is classified as a "partial failure", which means a local problem with the supply for a specific consumer. In the case when the supply of the product completely stops within the entire network, this is already considered a "full failure", which indicates a serious failure in the functioning of the TS.

Failure of a TS is closely related to the nature and scale of the consequences it causes. This means that determining the very fact of failure requires taking into account the impact on the network operation, as well as the level of service to consumers. To do this, during the analysis of the reliability of the TS, it is necessary to conduct a comprehensive assessment of the consequences, including both local failures that affect only individual consumers and global ones that disrupt the operation of the entire system. Such an assessment allows you to clarify which events or situations can be classified as a failure, taking into account the context of the study. This is especially important during multi-level studies, where each level requires its own criteria for determining failure, taking into account the specifics of the network operation, the tasks of its elements and the acceptable degree of risk.

Literature Review

The concepts of "reliability" and "economy" are often considered mutually contradictory for operating systems, since increasing reliability usually requires additional costs that can reduce economic efficiency [4, 6]. However, this relationship is not rigid: in certain cases, successful optimization or implementation of new technologies allows to achieve an increase in both parameters. For most systems, there is a certain level of reliability that can be considered optimal. Below this level, the growth of overall reliability slows down significantly even with significant improvement of individual elements [1, 2]. Further investments in such a situation lead to an unjustified increase in costs, which is not rational from an economic point of view. Therefore, it is important to consider the balance between economy and reliability and determine the limit beyond which additional costs cease to produce significant results. The framework for optimizing reliability is aimed at minimizing the costs associated with the modernization of objects, in particular stone building structures [8]. The unknown parameters of the finite element model that minimize the function of estimating the discrepancy between the experimental and numerical dynamic properties are integrated in a numerical method that allows finding the global minimum point and assessing its reliability [5]. In the article [11], a multilayer section strategy is presented for three-dimensional optimization of the location of discrete elements, which can be used to optimize and improve the reliability of technical systems. Various methods of structural reliability analysis include response surfaces and neural networks in optimizing the reliability of engineering systems [7].

The key characteristics that determine the quality of the TS structure are reliability and cost-effectiveness [2,4,6]. These properties are fundamental for assessing the efficiency of the network, since they affect its ability to meet the needs of consumers while minimizing resource costs and ensuring stable operation under various operating conditions.

Research Aim

The aim of the article is to develop a methodology for assessing and improving the reliability of vehicles by integrating approaches to modeling and cost optimization that take into account their technical and economic characteristics.

Discussion of Results

Increasing the reliability of any system usually requires additional financial costs. At the same time, there is a certain level of reliability, exceeding which becomes economically inexpedient, since further investment gives only a slight improvement. This creates a conflict between the desire for maximum reliability and the need to ensure cost-effectiveness, which complicates the development of optimal solutions for TS. For assessing the reliability of TS, an important parameter is the concept of "failure". However, this concept is multifaceted, since failure has different effects on consumers and the system as a whole. The close relationship between failure and its consequences requires careful analysis. It is necessary to clearly define the criteria for what exactly is considered a failure at different levels of research, as well as take into account the impact of such failures on the operation of the network and its users. This complicates the development of universal approaches to assessing and improving TS reliability.

By TS reliability we will understand the probability of failure-free operation of the system for a given time under the conditions of its normal functioning. This is a key parameter that characterizes the ability of the system to perform its functions without interruptions or failures.

To analyze this characteristic, the system structure reliability function is used, denoted as $R=f(r)$. It describes the dependence of the overall reliability of the TS R on the reliability of its individual elements r. This function takes into account the system configuration, in particular: the type of connection between the elements (serial, parallel or combined); the number of elements and their impact on the overall reliability; features of redundancy (for example, active or passive).

The dependence $R=f(r)$ allows you to model the behavior of the system depending on its components and helps to determine the optimal parameters to increase the reliability of the entire structure.

Equation

$$R = \sum_{i=1}^{n+1} A_i^{p-i+1} \qquad (1)$$

describes a polynomial reliability model for complex redundant technical systems with a combined structure [6]. In this equation, R is the overall reliability of the structure, r – is the reliability of each network section, p – is the total number of sections, n – is the number of rings in the structure, and A_i are the coefficients of the polynomial that determine the contribution of each member to the overall reliability. These coefficients depend on the topology of the system and the relationships between the elements.

The dependence takes into account various aspects of the structure's functioning. The degrees of r reflect the contribution of each section or its backup copy to the overall reliability of the system, while the coefficients A_i take into account the location, interaction of sections and rings in the structure, the method of redundancy, and the number of active elements.

The model is used to determine network parameters, optimize its configuration, and assess the impact of individual components on overall reliability. To use the equations, first determine the parameters p, n, r, and the network configuration to calculate the A_i coefficients. Then, substitute the reliability values of the sections r and calculate the overall reliability R. This model is particularly useful for the analysis and optimization of complex TS with rings and redundant paths. However, its limitations lie in the assumption of uniform reliability of the sections r. For heterogeneous systems, the equations must be adapted, and the A_i coefficients require detailed calculation for a specific topology.

The coefficients of the polynomial A_i determine the contribution of individual sections to the overall system reliability. Terms with lower degrees (r^{p-n+1}) have a smaller impact on R, reflecting the reduced influence of distant redundant rings on overall reliability in the case of element degradation.

The model has basic boundary conditions. If each section of the structure has reliability r=1, the overall system reliability must also be R=1. If the reliability of each section r =0, the overall system reliability must also be R=0, the reliability of all sections is the same. That is, $r \in (0;1)$, $R \in (0;1)$, $r_1 = r_2 = \ldots r_p = r$.

The coefficients of the polynomial A_i determine the contribution of sections to the system reliability. The terms with lower degrees (r^{p-n+1}) make a smaller contribution to R, which corresponds to the smaller influence of remote backup rings on the overall reliability in the event of element degradation.

In the case when the number of rings of the structure is equal to the number of sections (p=n), all sections have an equal impact on the overall reliability. In this case, the model reflects a structure with the maximum level of redundancy, where each section is critically important. If the structure is symmetrical (e.g., identical regions or rings), all A_i coefficients have the same value. This simplifies the model.

The limit point [6] $G(r_{gr}, R_{gr})$ is defined as a solution to the system of equations:

$$\begin{cases} R = f(r), \\ R = r \end{cases} \tag{2}$$

Substituting R=r into the function $f(r)$, we obtain:

$$f(r_{гр}) = r_{гр} \tag{3}$$

where: $f(r)$ is the structural reliability function, which describes the dependence of the overall reliability R on the reliability of the elements r; r_{gr} is the value of the reliability of the elements, at which the reliability of the system is equal to the reliability of the elements.

The limit point $G(r_{gr}, R_{gr})$ has the properties of the reserve balance and optimality. Under the condition $r \leq r_{gr}$, the reserve significantly increases the reliability of the structure R. When $r > r_{gr}$, the contribution of the reserve decreases, as the function $f(r)$ approaches the straight-line R=r and further increase in r becomes less effective.

The value of r_{gr} indicates the limit beyond which the reliability of the elements r becomes sufficient to ensure the effective functioning of the entire structure without excessive costs.

The function R=r is the bisector of the first quadrant, which describes the situation when the reliability of the structure and the reliability of its elements are identical. The expression R=$f(r)$ depending on the topology of the system has the form of a curve that lies above the straight-line R=r at $r < r_{gr}$ and can approach it at $r > r_{gr}$. The point of their intersection $G(r_{gr}, R_{gr})$ is key in the analysis.

The value of r_{gr} is determined by numerical or analytical solution of the equation $f(r)$=r. In the problem of system optimization, finding r_{gr} allows you to determine the level of reliability of elements that is optimal for ensuring the effective functioning of the system.

To determine the point G, we use the numerical algorithm [9,10] to find the root of the equation $g(r)$=0 on the interval $[a, b]$ where the function $g(r)$ changes sign. This means that $g(a)\cdot g(b) < 0$. We apply iterative division of the interval in half and checking the signs of the function until the interval becomes sufficiently small.

Input data: function $g(r)$=$f(r)-r$, where $f(r)$ is the reliability function of the structure; initial interval $[a, b]$, where $g(a)\cdot g(b) < 0$; given accuracy ε, which determines how close the solution will be to the root.

Let's consider the steps of the algorithm.

1. Check the initial interval: whether the condition is met:

$$g(a) \times g(b) < 0. \tag{4}$$

If the condition is not met, the method cannot be applied (the root may be missing, or the interval is chosen incorrectly).

2. To calculate the midpoint, we find the middle of the interval:

$$c = \frac{a+b}{2}. \tag{5}$$

3. We evaluate the value of the function at point c by calculating $g(c)$: if

$g(c)$=0, then point c is a root: $r_{gr} = c$;

$g(a)\cdot g(c)$<0, the root is in the left part of the interval $[a, c]$, so we update b=c;

$g(c)\cdot g(b)$<0, the root is in the right part of the interval $[c, b]$, so we update a=c.

4. Let's check the stopping criterion: is one of the conditions met:

- the length of the interval has become less than the specified accuracy: $|b-a| < \varepsilon$;

- the value of the function at the midpoint is close to zero: $|g(c)| < \delta$, where δ is the permissible error of the function.

If the criterion is met, point c is considered an approximate root: $r_{gr}=c$.

5. We continue dividing the interval and refining the solution until the specified accuracy is achieved.

The initial data of the algorithm is the value $r_{gr}=c$, which approximates the root with a given accuracy and the number of iterations required to achieve accuracy. The advantage of this method is guaranteed convergence. It always converges if $g(a)\cdot g(b) < 0$ and the function is without discontinuities on $[a, b]$, as well as simplicity of implementation. It is not necessary to calculate derivatives, but only the value of the function. However, the number of iterations in this method increases with the length of the initial interval and the root must lie only within the interval $[a, b]$.

The domain of the function $R=f(r)$, $r\in(0;1)$, the range of its values $R\in(0;1)$. The function strictly increases throughout the domain of definition in most practical cases: $f(r)\uparrow$: $r_1<r_2\rightarrow f(r_1) < f(r_2)$. It should be noted that the redundancy system must be constructed in such a way that adding or improving the reliability of elements always improves the operation of the entire system and the function $f(r)$ is continuous and has a non-zero derivative $f'(r)>0$ on the entire interval $r\in(0;1)$.

To assess the effectiveness of increasing the reliability of the TS structure in the context of a compromise between reliability and cost-effectiveness, it makes sense to use the ratio between the increase in system reliability (ΔR) and the increase in element reliability (Δr). Given that Δr is proportional to costs, the effectiveness can be formalized as:

$$E=\frac{\Delta R}{\Delta r}, \tag{6}$$

where: $\Delta R=f(r+\Delta r)-f(r)$ – the increase in system reliability due to an increase in element reliability by Δr; and Δr is the increase in element reliability proportional to the additional costs. The ratio $\Delta R/\Delta r$ (6) shows how significantly the increase in element reliability Δr affects the overall system reliability ΔR. The larger the value of E, the more effective the increase in reliability.

Efficiency analysis indicates a dependence on the shape of the function $f(r)$. If the function $f(r)$ is concave ($f''(r)<0$), then ΔR decreases with increasing r, i.e., the efficiency E decreases at high levels of element reliability. In the case when the function $f(r)$ is convex $f''(r)>0$, the efficiency E can increase with increasing r.

For small changes in Δr, the derivative of the function can be used:

$$E=\frac{\Delta R}{\Delta r}\approx\frac{f'(r)}{1}, \tag{7}$$

where $f'(r)$ – the rate of change of system reliability relative to the reliability of the elements.

To find the optimal value of r that provides the highest efficiency, the equation is solved:

$$\frac{\partial E}{\partial r}=0, \tag{8}$$

which is equivalent to maximizing the value of E.

Increasing the reliability of system elements makes sense if the ratio $E=\Delta R/\Delta r$ is sufficiently large. To make investment decisions on increasing the reliability of elements, it is worth considering the current level of reliability r, the function $f(r)$, which describes the redundant system, and the additional costs proportional to Δr.

A promising direction of research is to simplify reliability models by approximating complex polynomials by simpler functions, which will reduce the complexity of the analysis. In economic optimization, it is important to model the costs of increasing reliability and find trade-offs between costs and efficiency. Theoretical research can include the analysis of reliability limits, the expansion of mathematical approaches, and their formalization through optimization methods.

In stone structures that have the features of technical systems, modeling, reliability analysis, numerical methods, and optimization can be used, which will allow increasing reliability by identifying critical areas for reinforcement. Reliability is determined by the stability of blocks, mortar layers, or joint zones. Damage zoning allows for the identification of critical areas for reinforcement, and combined models take into account different types of loads and failures. The overall stability can be described as a function of the reliability of individual components. Achieving a balance between reliability and cost requires a systems approach that considers the behavior of the structure through the function $R=f(r)$ and the efficiency of investment ($\Delta R/\Delta r$).

Conclusions

Achieving a high level of TS reliability requires a systematic approach that combines the analysis of structural elements, their impact on the overall network reliability and optimization of resource costs. It is important to find a balance between the level of reliability and cost minimization.

The ratio of the increase in system reliability ΔR to the increase in element reliability Δr determines the efficiency of investing in the system. Efficiency decreases at high levels of reliability due to a decrease in the effect of redundancy.

The limiting point $G(r_{gr}, R_{gr})$ determines the balance between the reliability of the elements and the system. When this point is exceeded, additional costs for increasing reliability become ineffective.

Simplifying reliability models by approximating complex polynomials will reduce the complexity of the analysis. Economic optimization is aimed at modeling costs and trade-offs between reliability and cost. Building structures (in particular, stone) have features of technical systems with prospects for modeling, optimization to increase reliability through analysis of the stability of blocks or connection zones. Damage zoning in them allows to identify critical areas for reinforcement, and combined models take into account different types of loads. Reliability models can be developed to assess the stability of structures.

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ОПТИМІЗАЦІЯ НАДІЙНОСТІ ТЕХНІЧНОЇ СИСТЕМИ З УРАХУВАННЯМ МІНІМІЗАЦІЇ РЕСУРСІВ

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У статті розглядається питання оптимізації надійності резервованих технічних систем шляхом комплексного підходу, що об'єднує теоретичне моделювання, чисельні методи аналізу та економічну оптимізацію. Основна увага приділяється забезпеченню балансу між надійністю системи та витратами на її вдосконалення. Такий підхід спрямований на досягнення стійкої та ефективної роботи технічних систем *навіть за наявності зовнішніх впливів або відмов окремих компонентів. Теоретична частина дослідження ґрунтується на аналізі функції надійності, яка описує залежність загальної надійності системи від характеристик її елементів. Важливим аспектом цього аналізу є виявлення критичних компонентів, які мають найбільший вплив на функціональність системи. На основі цього визначаються оптимальні стратегії резервування, що дозволяють підвищити стійкість системи до відмов. Введення критичних значень надійності дозволяє оцінити ефективність додаткових інвестицій у покращення окремих елементів. Зокрема, за певним рівнем надійності приріст ефективності від подальших вкладень значно зменшується, що потребує оптимального підходу до розподілу ресурсів.*

Економічна складова статті зосереджена на розробці стратегій рентабельного управління витратами, пов’язаними з підвищенням надійності технічних систем. Це включає аналіз витрат на вибір матеріалів, структурне посилення елементів конструкції або створення резервних компонентів. Оптимальні рішення базуються на моделюванні витрат і вигод для різних сценаріїв функціонування системи, що дозволяє мінімізувати ресурси без шкоди для надійності. Чисельні методи використовуються для точного визначення критичних зон у складних конструкціях. Наприклад, у кам'яних конструкціях такі методи дозволяють виявляти місця поступового руйнування, які потребують посилення. Завдяки цьому можна здійснювати цілеспрямоване зміцнення конструкцій, підвищуючи їх стійкість до зовнішніх навантажень і зношення. Комбіновані моделі враховують різні типи навантажень і можливі сценарії відмов, що дозволяє інженерам приймати обґрунтовані рішення під час проєктування і модернізації об’єктів.

Перспективи подальших досліджень у цій сфері пов’язані з удосконаленням чисельних методів аналізу, а також проведенням експериментальних випробувань для верифікації розроблених моделей. Крім того, розширення застосовності інтегрованого підходу до різних типів технічних систем і умов експлуатації дозволить підвищити ефективність і надійність об’єктів інженерії та будівництва на національному й міжнародному рівнях. Зокрема, інтеграція економічної та технічної оптимізації є ключем до створення стійких систем, які відповідають сучасним вимогам безпеки та ефективності.

Ключові слова: *структурна надійність системи, ресурсна ефективність, резервування.*